THE IMAGINATIVE MATHEMATICS OF BLOCH'S *UNIMAGINABLE MATHEMATICS OF BORGES'S LIBRARY OF BABEL*

Curtis Tuckey

In his charming and thought-provoking little book, *The Unimaginable Mathematics of Borges's Library of Babel*, William Goldbloom Bloch, a Berkeley-trained mathematician and professor of mathematics at Wheaton College, in Massachusetts, has taken Jorge Luis Borges's seven-page Kafkaesque fantasy about a vast library containing all possible books and put together a small volume of mathematical variations, improvisations, and extrapolations. He is delighted to share his musings with anyone who cares to follow along, and his enthusiasm is infectious. He says,

> for those of mathematical bent who've not read Borges, I hope this volume inspires two things: a desire to explore more of Borges's work. . . and, equally, a desire to learn more about the math tools I employ. . . [T]here are three main themes woven into this book. The first one digs into the Library, peels back layers uncovering nifty ideas, and then runs with them for a while. The second thread is found mostly in the "Math Aftermath" sections [in which] I develop the mathematics behind the ideas to a greater degree. . . . The third focus is on literary aspects of the story and Borges. . . . [M]y purpose is to make explicit a number of mathematical ideas inherent in the story. . . I am a tour guide through a labyrinth . . . Besides enhancing the story, the reader's reward will be an exposure to some intriguing and entrancing mathematical ideas. [xiii-xiv]

There is little focus on "literary aspects" of the story, but once Bloch's mind latches onto a "nifty idea" he lets his mathematical imagination run freely. Nifty ideas include: the number of books in the Library of Babel and how unimaginably large it is; the possibility or impossibility of a catalog of the collection; mathematical models of a book with infinitely many pages; alternative physical geometries for the universe of the Library; possible floor plans for the hexagonal galleries, and surprising properties of paths through the labyrinth. Robert Burton's exhortation in *Anatomy of Melancholy*, quoted by Borges at the beginning of "The Library of Babel," is equally appropriate here: "In this way you shall contemplate the variations of the twenty-three letters." Indeed, several of Bloch's variations on a theme of Borges originally appeared in this journal (see "The Unimaginable").

The *Unimaginable Mathematics of Borges's Library of Babel* garnered honorable mention in mathematics for the 2008 American Publishers Awards for Professional and Scholarly Excellence (the PROSE Awards), and deservedly so. Reviewers, professional and amateur, have been uniformly delighted by Bloch's mathematical tour through the Borgesian labyrinth. Nonetheless, I think it worth looking further into some of the mathematics behind Bloch's conjectures and conclusions, with particular emphasis on the chapters called "Information Theory" and "Real Analysis."

THE LIBRARY OF BABEL

In "The Library of Babel" (1941), Borges described a mythical-religious-fantastical universe consisting of an impossibly vast honeycomb of book-filled galleries that have existed for all time. The ceilings of the galleries are hardly taller than the height of a normal librarian, the light is insufficient (and unceasing), the librarians are forced to sleep standing up in tiny cubicles, and the very low railings around the vast, bottomless, ventilation shafts furnish a constant source of mortal danger. Moreover, the librarians are surrounded by an incomprehensibly enormous collection of books, almost all of which are unsearchable, unclassifiable, and unreadable, but among which are included all possible combinations of the letters of the alphabet—"all that is able to be expressed, in every language."

> *All*—the detailed history of the future, the autobiographies of the archangels, the faithful catalog of the Library, thousands and thousands of false catalogs, the proof of the falsity of those false catalogs, a proof of the falsity of the true catalog, the gnostic gospel of Basilides, the commentary upon that gospel, the commentary on the commentary upon that gospel, the

> true story of your death, the translation of every book into every language, the interpolations of every book in all books, the treatise that Bede could have written (but did not) on the mythology of the Saxon people, the lost books of Tacitus. (*Collected* 7)

The idea of a physical library of all possible books has a long history, but calculating the *size* of such a collection had to wait for the development, in the seventeenth century, of methodical techniques for generating permutations and combinations. In 1622 the Swiss mathematician and astronomer Paul Guldin determined that if all possible combinations of between three and twenty-three letters were printed sixty characters to the line and one-hundred lines to the page, bound into books one-thousand pages long, and housed in cubical warehouses 432 feet on a side, then the space required to accommodate these warehouses would exhaust the available surface of the globe. In 1636 the Jesuit music theorist and mathematician Marin Mersenne performed a similar calculation in *Harmonie universelle*, his monumental work on musical instruments and the theory of music. There he discussed the impracticality of publishing an encyclopedia of all 1,124,000,727,777,607,680,000 permutations of just twenty-two musical notes: if 720 permutations were printed on each sheet and if each ream of paper were compressed to a thickness of one inch, the height of such a stack would well exceed then-current estimates for the distance between the earth and the stars. (On the other hand, Mersenne did manage to publish an encyclopedia of all 40,320 permutations of eight notes, on 672 folio pages (see Eco and Knuth)

In his 1939 essay "The Total Library" Borges traced the evolution of the idea of a library of all possible books from ancient arguments about whether chance could generate order from disorder (Leucippus, Democritus, Aristotle, Cicero), through the conception that knowledge could be created by mechanically combining properties, logical propositions, or words (Llull, Pascal, Swift) to the idea that all possible works of literature could be found among all possible combinations of letters and spaces (Gustav Fechner, Lewis Carroll, Kurd Lasswitz, Theodor Wolff). Because of its inclusion in Clifton Fadiman's perennially popular mathematical anthology, *Fantasia Mathematica* (1958), Lasswitz's relatively obscure and seemingly unremarkable story, "The Universal Library" is perhaps more familiar to American mathematicians than Borges's world-famous "Library of Babel." The professor, sitting in his armchair and smoking his pipe, pencil and paper handy, egged on in his mathematical musings by his wife and his publisher, calmly calculates the number of books one can create using one-hundred characters (upper- and lower-case letters, punctuation marks, spaces, and other special characters) if one has 500 pages per book, 40 lines per page, and 50 characters per line. (His answer: $10^{2,000,000}$.)

To the contrary, Borges was keen on emphasizing the mystical and the fantastic. As he wrote in "The Total Library," "I have tried to rescue from oblivion a subaltern horror: the vast, contradictory Library, whose vertical wildernesses of books affirm, deny, and confuse everything like a delirious god" (*Selected* 216).

Rather than performing a computation and studying the resultant magnitude as a mathematician or astronomer might, Borges instead delivered a darkly comic vision of a librarian's hell. And whereas Lasswitz's description of the physical parameters of the books is everywhere precise, Borges confounds a determination of the total number of books in his Library by being vague in crucial spots. Very precisely, Borges says that each hexagonal gallery contains five bookshelves of thirty-two books on each of four walls; each book contains four hundred ten pages; each page, forty lines. There are exactly twenty-five orthographic symbols: "the twenty-two letters of the alphabet," the comma, the period, and the space. But then, vaguely, each line of each book contains "approximately eighty black letters," and there are an unspecified number of letters printed on the spines of the books.

However, if we tidy up Borges's description by assuming that there are *exactly* eighty characters and spaces per line and if we ignore any extra characters printed on the spine, then each book will contain 80 characters per line times 40 lines per page times 410 pages per book, for a total of 1,312,000 characters per book. Each character has twenty-five possibilities and all possibilities must be realized, so there are a total of $25^{1,312,000}$ books. It is with the determination of this large number that William Goldbloom Bloch begins his imaginative book.

Bloch's discussion parallels Lasswitz's, but with purpler prose. Concerning the number of books containing sixteen occurrences of the letter *h* among "an otherwise uniform desert of the letter *g*," Bloch writes,

> These books, droning wearily of *g*, with a little respite provided only by the scant sixteen instances of *h*, are not typographical phantasmagoria to inflame the imagination or addle the senses, and yet if they were all collected into a subsection of the Library, they would occupy a space greater than three times our known universe. (21)

Bloch concludes his initial chapter with a succinct, characteristic theme: "*The number of books in the Library, though easily notated, is unimaginable*" (22).

"INFORMATION THEORY" (CATALOGS OF THE COLLECTION)

Bloch notes that if one considers a library catalog to be a collection of descriptions of books, and if one limits oneself to, say, half-page descriptions, in the alphabet of the Library, then it is impossible to provide a unique catalog entry for each book: there are $25^{1,312,000}$ books but only $25^{1,600}$ unique descriptions. From this calculation Bloch concludes that "*The Library is its own Catalog. Any other catalog is unthinkable*" (39). His conclusion is correct in an important sense, but he draws it too quickly, and his chapter, though well written and thought provoking, lacks both a unifying theme and a connection with the science of information theory.

Mathematically, let us identify a book in the Library with a character string of length 1,312,000, in the 25-character alphabet of the Library, and let us define a *description* of a book to be any character string (again, in the character set of the library) of finite length. This notion encompasses written descriptions *of any kind whatsoever*, in any language. A description of a book might be a computer program that has precisely that book as its output, or directions on how to locate that book in the Library of Babel (in terms of floor, gallery number, shelf, and position), or a mystical encoding only understood by some oracle. The following information-theoretic result ties Bloch's ideas together and answers his conjectures.

Theorem. *For any way of assigning unique descriptions to books in the Library, most of the descriptions (more than 95%) will be at least as long as a book.*

This does not quite demonstrate that any other catalog is "unthinkable," but it does show that if a catalog is a collection of descriptions of books, then one can hardly do better than by simply listing the complete contents of each and every one of them.

The information-theoretic concept of the *Kolmogorov complexity* of a character string is defined to be the length of (one of) the shortest computer program(s), written in some fixed universal programming language, that has the given string as its output. If the shortest program to output a given character string is no shorter than the string itself, the string is said to be *incompressible*. The theorem says that most of the books in the Library are incompressible (in addition to being incomprehensible).

In a related question, Bloch considers compact notations for large numbers, such as $2^{4,781}$ (which if written in ordinary decimal notation would be a much lengthier 1,440 digits long) and remarks that one may "legitimately wonder if every integer might have a remarkably condensed form" (37). Bloch does not make a strong argument either way, and his inconclusive discussion of "the median of the prime numbers expressible in 100 digits" is a red herring. The answer to the question is a definite *no*: most character strings, made from a fixed set of characters, are incompressible (or very nearly so). However we might try to describe numbers in condensed form, if we are restricted to n symbols, then almost all of their numerical descriptions can hardly be more condensed than their representation in base n.[1]

"Real Analysis" (Modeling Infinite Books)

In a footnote (first included in the edition of 1944) attached to the last word of the last sentence of "The Library of Babel," Borges, in the voice of

1 The theorem is easily demonstrated by a counting argument. For each natural number n, there are 25^n descriptions of length n. From this one can determine that the number of descriptions shorter than a book (of 1,312,000 characters) is given by the sum, $25^0 + 25^1 + 25^2 + \cdots + 25^{1,312,000-1}$. Even if one has not seen a geometric sum before one can easily verify using algebra that the product $(x^0 + x^1 + x^2 + \cdots + x^{n-1})(x - 1)$ telescopes to $x^n - 1$, and hence that for x not equal to 1, the sum $x^0 + x^1 + x^2 + \cdots + x^{n-1}$ is equal to the fraction $(x^n - 1)/(x - 1)$. Taking $x = 25$ and $n = 1,312,000$, we see that the number of descriptions shorter than a book is equal to $(25^{1,312,000} - 1)/24$, or just a bit more than 4% of the size of the entire Library. This implies that more than 95% of the descriptions of books would have to be at least as long as a book. A version of the argument can be found in any textbook on information theory. See, for example, Li and Vitani 109.

By a similar argument one can conclude that for any assignment of descriptions to any m distinct items, most of the descriptions will have to have at least as many as $[\log_{25} m]$ characters. In particular, for any kind of character-string representation for the numbers 0 through $m - 1$, most of them will have to have at least $[\log_{25} m]$ characters—in other words, most of them can be no more compressed in length than they are in ordinary base-25 place-holder notation. Better results are available, but these suffice for the questions at hand.

the editor of the manuscript, relates the following observation of a personal friend of his:

> Letizia Álvarez de Toledo has observed that the vast Library is pointless: strictly speaking, all that is required is a single volume, of the common size, printed in nine- or ten-point type, that would consist of an infinite number of infinitely thin leaves. (In the early seventeenth century, Cavalieri said that all solid bodies are the superposition of an infinite number of planes.) Using that silken vademecum would not be easy: each apparent page would open into other, similar pages; the inconceivable middle page would have no "back." (*Collected* 118)

A *vademecum* is a book for ready reference, or a pocket handbook, from the Latin for "go with me." Here in one volume one could carry, Kindle-like, all possible books for ready reference at any time!

Bloch says, "The mathematical analysis of [such a book] hinges on what is meant by the phrase 'infinitely thin pages.' Three different interpretations of 'infinitely thin' lead to three Books similar in spirit, but disparate in the details" (46). After presenting his three interpretations (the first of which does not have infinitely thin leaves) Bloch concludes that "*Regardless of which interpretation we assume, if the pages are 'infinitely thin,' then by necessity the [book] itself is infinitely thin*" (54). This is a spectacularly surprising result, but it is by turns misleading and mistaken. Briefly, here are the problems with Bloch's analysis.

In his second interpretation, Bloch (as have many other commentators) models infinitely thin leaves as rectangular sections of a Euclidean plane, and models an infinite book as a box-like collection of infinitely many such leaves. For example, one can imagine such a "book" as being made up of a countably infinite collection of six-by-nine inch "leaves" lying parallel to the *xy*-plane and projecting rigidly from each rational point in the unit interval of the *z*-axis. Uncontent with merely saying that the leaves have no thickness, Bloch takes several pages to explain the point-set definition of *measure zero* and then *defines* "infinitely thin" to mean "having measure zero." This doesn't help in the least in elucidating what it means for a leaf to be "infinitely thin"—planes have zero thickness and are rather uncontroversially infinitely thin already. And it actually confuses the issue when Bloch tries to explain how thick a book of such pages would be. Having *defined* "infinitely thin" to mean "having measure zero" he concludes that since a countably infinite collection of such leaves would have to be of measure zero, the book itself would, *by his definition*, have to be infinitely thin. "In other words," he says, "if we looked at the Book sideways, we would not be able to see it, let alone open it." This conclusion is misleading because for geometrical structures "measure" does not correspond to spatial extension. It is certainly possible to have a countable infinity of pages densely packed into a book of one inch in thickness, or any other thickness. (For a real-world comparison, by the *volume* of a physical book, we mean the volume of the extended structure: width times height times thickness. We do not mean the sum of the volumes of the component electrons, neutrons, and protons, which science tells us make up less than 0.00000000000001% of the spatial volume of a physical structure.) Perhaps one reason for the confusion is that, unlike the pages of an actual book, planar sections cannot be stacked one atop the next.[2]

In his third interpretation Bloch defines "infinitely thin" in terms of the infinitesimals of Robinsonian analysis (nonstandard analysis), but he mistakenly assumes that an infinite sum of infinitesimals must itself be infinitesimal. He writes:

> By the rules of nonstandard analysis, we compute the thickness of the Book by adding together all of the infinitesimals. For a summation such as this one, adding the infinite number of infinitesimals produces yet another infinitesimal, so the book is, again, infinitely thin: never to be seen, never to be found, never to be opened. (54)

2 The Italian mathematician Bonaventura Cavalieri (1598–1647) arrived at his eponymous principle and many other important theorems by imagining geometrical solids to be *book like*: not consisting of plane regions of zero thickness, with another plane between any given two, but instead consisting of infinitely many "equidistant and parallel indivisibles," which are infinitely thin but nevertheless of nonzero thickness. He said that "a line is made up of points as a string is of beads, a plane is made up of lines as a cloth is of threads, and a solid is made up of plane areas *as a book is made up of pages*" (Kline 349, my emphasis). Cavalieri's intuitive picture is not supported by the standard model of Cartesian space, however. It is a consequence of what is known as the Bolzano-Weierstrass Theorem, not proved until the late nineteenth century, that in a book with infinitely many pages bound between finite covers, there must be at least one "leaf of convergence" having the property that between that leaf of convergence and any other leaf, there is always yet another leaf. If having one such frightful leaf were not enough, if our model has a leaf sewn in at each rational point in its spine, then *every* leaf is a leaf of convergence.

This is not the case. In Robinsonian analysis it is perfectly reasonable to take N to be an infinite hypernatural number, so that $1/N$ is infinitesimal, and consider an ideal book consisting of N leaves of thickness $1/N$. The thickness of such a book is given by the infinite sum of the thicknesses of the leaves, and that sum is *exactly* equal to 1. Not infinitesimal, not infinite: 1.

$$\underbrace{1/N + 1/N + \ldots + 1/N}_{N \text{ terms}} = 1$$

A possible source for the error is perhaps that in *standard* analysis *there are no* infinite sums, and instead one must make do with limits of sequences of finite partial sums. The infinite sums of nonstandard analysis are most definitely not limits of finite partial sums.[3]

THE OTHER CHAPTERS

In Chapter 4, Bloch uses the physical universe of the Library as a foil for a lengthy discussion (a full quarter of the book's exposition) of popular topics from elementary topology: Möbius bands, Klein bottles, and higher dimensional analogs of spheres and toruses. (Incidentally, these topics are also well represented in *Fantasia Mathematica*.) The main inspiration for this chapter, Borges's statement that "*the Library is a sphere whose exact center is any hexagon and whose circumference is unattainable*," is a metaphor with a long history and many interpretations—including a vision of infi-

3 In this footnote to a commentary on a commentary on a footnote (a delightfully construction worthy of Borges) I note that although Bloch (and several other commentators) believe that "the book proposed in the final footnote of 'The Library Babel' is of a similar structure to the book described in Borges's short story, 'The Book of Sand' " (*The Unimaginable* 131), this latter book is manifestly different in structure from any of the models that Bloch (or any of the other commentators) have described. The book in "The Book of Sand" has literally infinitely many pages bound into a book of ordinary thickness in such a way that (a) every page has a next page and a previous page, and (b) there is no first page or last page. Comparing the Book of Sand with the two models discussed above, we have the following situation: a countably infinite book with a leaf at each rational number strictly between 0 and 1 will satisfy (b) but not (a), while the hyperfinite book with N leaves of thickness $1/N$ will satisfy (a) but not (b). It is not very hard to construct a Robinsonian model of the book described in "The Book of Sand," but I shall leave that for a later commentary on a footnote to a commentary on a commentary on a footnote.

nite Euclidean space; see Borges's essay "Pascal's Sphere" (*Selected* 351-53). But it is nonetheless fun to think about what other spaces might be available for housing the Library. Perhaps following a suggestion of Einstein's for our own universe, Bloch proposes in this lengthy chapter that a reasonable model for the space that houses the Library is the 3-dimensional boundary of the 4-dimensional sphere.

In Chapter Five, "Graph Theory," Bloch considers possible floor plans for the Library of Babel. He does not come up with a layout that satisfies all of Borges's requirements at the same time, but this in itself is hardly damning. As with the description of the parameters of the books (characters per line, lines per page, pages per book), Borges's description of the layout of the Library is a mixture of specificity and confounding vagueness, and Borges himself adjusted the architecture subtly but significantly (and not entirely successfully) when he edited the original story of 1941 for a new edition in 1958. Bloch's discussion is interesting and original, but he does not take into account the substantial contributions of Antonio Toca in "La arquitectura de Jorge Luis Borges" (see also his article "La biblioteca de Babel" for more references) and Cristina Grau in *Borges y la arquitectura* (1989). Neither Toca nor Grau manage to satisfy all of Borges's requirements to the letter either, but Toca's elegant architectural renderings are especially compelling. And although Bloch would like to inject some mathematics into his discussion, when he finally does come to a mathematical conjecture (his "Conjecture of Extreme Disconsolation") he misstates the hypotheses in a way that makes the conclusion false, and "despite not offering much of a framework for the proof" because it would be "too messy to offer up in these pages," he leaves the proof as an exercise for the reader. The proof is not quite small enough to fit into this margin, but a sketch of a proof easily fits into a footnote.[4]

4 Bloch's Conjecture is not quite true, as stated: he should probably have said that *either* there are some galleries that are completely inaccessible to others *or* there will have to be wall-sharing galleries arbitrarily large finite walking distances apart. Let us assume that all of the galleries are mutually accessible. In this case, there is essentially one long, winding path or hallway that linearly orders the galleries in the plane, and in particular, there is a unique path between any two given galleries. From some arbitrarily chosen gallery, let us have two librarians say "I may be some time," leave that gallery by opposite doors, visit a new gallery on that floor in each time unit, and never turn back. At any given time there will be a collection of galleries that have been visited at that time and another collection of galleries that are adjacent to (i.e., sharing a wall with) at least

In Chapter 6, Bloch discusses his idea of a "Grand Pattern" for the Library of Babel. This involves filling all available space with permutations of the set of all permutations of the set of all permutations . . . of books on bookshelves. While his discussion is nightmarish enough to be called Borgesian, there is too much hand-waving for it to be mathematically convincing. In Chapter 7 he draws a superficial (though poetic, and perhaps Borgesian) analogy between Turing machines and the human (or librarian) condition, saying, "The librarian's life and the Library together embody a Turing machine, running an unimaginable program whose output can only be interpreted by a godlike external observer" (125).

In Chapter 8 he gives a thoughtful review of some of what other critics have said in English about Borges and mathematics, but neglects everything written in Spanish, including the very relevant *Borges y la matemática* (2003) by Guillermo Martínez, and several articles in *Borges y la ciencia* (1999). Alberto Castellón's lengthy essay "Letizia, Borges y el infinito" (2008) appeared in the same year as *The Unimaginable Mathematics of Borges's Library of Babel*, and makes many of the same points, but perhaps appeared too late to get Bloch's attention (and vice versa).

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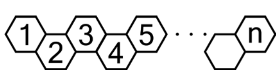
one of the visited galleries, but which have not yet been visited. The trick is to show that when the collection of visited galleries has reached a certain size, there will be so many unvisited adjacent galleries on its perimeter that the librarians will necessarily take a long time to reach all of them in their continuing journey. One way to do this is to consider any collection of hexagons (not necessarily having connecting vestibules) that form a zigzag of length *n*:



By our assumption that all of the galleries are mutually accessible by some path, each of these hexagons will be visited by one or other of the traveling librarians at some future time: i.e., there will be some time t_n at which all *n* of them have been visited. The key to the proof of the conjecture is that, at that time, there will be at least 2*n* galleries that are adjacent to already-visited galleries but which have not as yet been visited. (From each of the *n* hexagons in the zigzag, look along the lines both north and south until you find the first adjacent, unvisited gallery in each direction.) From time t_n it will take at least *n* more time units until all of those adjacent, unvisited galleries will be visited. Of those 2*n* galleries, let *B* be the *last* one that is eventually visited. (If there are two "lasts" then pick either one.) Then *B* has a face-sharing gallery *A* that had already been visited by one of the librarians (it does not matter which one) at time t_n, and it has taken one of the librarians (it does not matter which one) at least *n* steps to get to *B* starting at time t_n. Hence the (unique) path from *A* to *B* must be longer than *n*. Finally, we may take *n*, the length of the zigzag we started with, to be as large as we like.

At the end of the book, Bloch talks about the wear patterns and annotations in the mathematical books in Borges's personal library, now part of the National Library of Argentina, and reminds us of Copernicus's dictum, *mathemata mathematicis scribuntur* (mathematics is written for mathematicians), which he found inscribed in one of those books. He concludes, "It is my hope that this book belies that sentiment."

POSTSCRIPT

Bloch prefaced his book with a question, "Who is the intended audience for this work?", and his own initial response (whether tongue in cheek I cannot quite tell) was Umberto Eco. When thinking of the *type* of person whom one might write such a book for, the late Clifton Fadiman comes immediately to my mind. At Willy Ley's suggestion Fadiman included Lasswitz's "Universal Library" in *Fantasia Mathematica* in 1958, and as chief editor of the Book-of-the-Month Club, he published Borges's *Labyrinths* (which includes "The Library of Babel") in 1964. Twenty years later Fadiman recommended *Labyrinths* in his *Lifetime Reading Plan*, specifically mentioning the Library of Babel: "Borges plays with ideas," he said, "like a magician with his props, but the magic is more than legerdemain. His Library of Babel is also a universe and an emblem of infinity. . ." (139). Although he was not a mathematician, Fadiman was not afraid of mathematics. In the introduction to *Fantasia Mathematica*, he wrote,

> *I still know no real mathematics*, but I know what mathematics is about, and something of the way the mathematical mind seems to work, and a fair amount about the lives of the great mathematicians. I have written elsewhere, in an essay called "Meditations of a Mathematical Moron," about the pleasures of reading about mathematics, pleasures open to anyone who has received a conventional secondary-school education, and is willing to do a little mental work. (xiv, my emphasis)

In his review of *The Unimaginable Mathematics* in the *New York Sun* (September 24, 2008) Alberto Manguel wrote (italics added),

> *Though I confess that my mathematical illiteracy* made it difficult for me to follow many of his formulas and graphs, the lucidity of Mr. Bloch's arguments enlightened me nevertheless . . .

> Mr. Bloch notes in his preface that the ideal reader of his book is Umberto Eco. Unworthy as I am to aspire to the condition of the great polymath (with whom I share nothing but the girth and the beard), I was delighted and instructed by the book's wit and wisdom, and grateful for the guided tour through the mathematical foundations on which both the Library of Babel and its mirror, our universe, are so delicately balanced. (24)

When asked in 1983 whether there was a kind of kinship between poets and mathematicians, Borges himself replied,

> *My mathematics is very slight.* But I have read and reread Bertrand Russell. And I think there should be a kinship. And I suppose there is. There is a kinship between all things, especially between poets and mathematicians, and poets and philosophers, who are a measure of poets, I should say. (*Borges, the Poet* 83, my emphasis)

In his preface Bloch went on to say that the potential audience for his book should include mathematics-friendly Borges aficionados, together with Borges aficionados *to be*. In this latter class I would include young science-fiction and fantasy fans who possess a certain fearlessness for mathematical ideas. After all, exponentials and elementary combinatorics are already topics of grade-school mathematics. An example: in a 1996 TV spot produced by the Children's Television Workshop (*Sesame Street*), aimed at fourth- to sixth-grade schoolchildren, Dweezil Zappa, in a minute and twenty-one seconds, used combinatorics ("combina-*what?*") to calculate how many different outfits he could put together from a certain number of pants, shirts, and sweaters. From this calculation it is only a few steps in magnitude, not conception, to computing the number of books in the Library of Babel.

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